in our Curve, the greatest Breadth is when the Point F *divides the Line* A B *in extream and mean Proportion: whereas in the* Foliate *it is when* A B *is triple in power to* B F. *And the greatest* E F *or Ordinate in the* Foliate *is to that of our Curve nearly as* 3 *to* 4, *or exactly as* $\sqrt{\frac{2}{3}\sqrt{\frac{1}{3}}-\frac{1}{3}}$ *to* $\sqrt{5\sqrt{\frac{5}{4}}-5\frac{1}{2}}$.

But still these Differences are not enough to make them two distinct Species, *they being both defined by a like Equation, if the* Asymptote S G P *be taken for the Diameter. And they are both comprehended under the* fortieth Kind *of the* Curves *of the* third Order, *as they stand enumerated by Sir* Isaac Newton, *in his incomparable Treatise on that Subject.*

IV. *An easy Mechanical Way to divide the Nautical Meridian Line in* Mercator's *Projection; with an Account of the Relation of the same Meridian Line to the* Curva Catenaria. *By* J. Perks, M. A.

THE most useful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd *Mercator*'s; tho' its true Nature and Construction is said to be first demonstrated by our Countryman Mr. *Wright*, in his *Correction of the Errors in Navigation*. In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous,) but the Minutes and Degrees (or strictly, the *Fluxions of the Meridian,*) at every several Latitude are proportional to their respective *Secants*. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the same Latitude is to *Radius*.

The

The Reason of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection, the Meridians being all parallel, a Degree of Longitude at (suppose) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but *half* as much, the Radius of the Parallel of 60 Deg. (that is its *Cosine*) being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to those of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Cosine.*

In *Fig.* 3. let the Radius CD represent half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant: then is CS equal to its *Cosine* : and $CS : CM :: CD (= CM) : CE$, that is, as Cosine : to Radius :: so is Radius : to Secant. The Cosines being then, in this Projection, suppos'd all equal to Radius, or (which comes to the same) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude, (by the precedent Analogy) is supposed equal to the Secant of Latitude ; and consequently the Elements (Minutes, *&c.*) of the Meridian must be proportional to their respective Secants,

The Way Mr. *Wright* takes for making his Table of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and so proceeding to 89 Deg. Dr. *Wallis* (in *Phil. Transf.* No. 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$ &c. which gives the *Merid. part* required. How to find the same Mechanically by means of an easily-constructed Curve Line, is what I shall now shew.

I.

1. Prepare a Rular AB (*Fig.* 2.) of a convenient Length, in which let Bo be equal to the Radius of the intended Projection. To the Point o as a Center (on the narrower Edge of the Rular) fasten a little Plate-Wheel wh tight to the Rular, and of a Diameter a little more than the thickness of the Rular. Let KR (*Fig.* 3.) represent another long Rular, to which AR is a perpendicular Line. Place the Rular AB upon the Line AR, with the Center of the Wheel at A. Then with one Hand holding fast the Rular KR, with the other Hand slide the end B of the Rular AB by the Edge of KR; so will the little Wheel wh describe on the Paper a Curve Line ACB, to be continued as far as is convenient.

2. Having drawn the Curve ACB, draw a streight Line KR by the Edge of the Rular KR: which Line is the *Meridian* to be divided, and also an Asymptote to the Curve ACB.

3. In this Meridian, (accounting R to be the Point of its Intersection with the Equator,) the Point answering to any Degree of Latitude is thus found. In the perpendicular AR, make RG equal to the *Cosine* of Latitude (Radius being AR,) and from G draw GC parallel to KR, and intersecting the Curve in C. With Center C and Radius $CM = AR$, strike an Arc cutting the Meridian at M; so is M the Point desir'd.

4. In the Curve AC, let c be a Point infinitely near to C, and cm, $(= CM,)$ a Tangent to the Curve at c, making the little Angle MCm, to which let the Angle RAr be equal: so is $Rr = Md$ (a Perpendicular from M to cm.) Draw CD equal and parallel to AR, intersecting KR in S. With Center C and Radius CD draw the Arc DM, and its Tangent DE and Secant CE.

5. Because of the like Triangles CDE, Mdm; $CD : CE :: Md : Mm$, that is, as Radius to Secant of the Arc DM, (whose Cosine is $CS = GR$,) :: so is Md

($= Rr$ a Degree or Particle of the Equator :) to Mm the Fluxion or correſpondent Particle of the Meridian Line RM. Whence, and from what is premiſed concerning the Nature of this Nautical Projection, 'tis evident that RM is the *meridional Part* anſwering to the Latitude whoſe Coſine is GR. Or thus; With Center R and Radius AR deſcribe the Quadrant $A\varkappa\alpha$, in which let the Arc $A\varkappa$ be equal to the given Lat. From $\varkappa$ draw $\varkappa C$ parallel to KR, and interſecting the Curve in C, ſo is $C\varkappa$ the Meridional Part deſir'd being equal to RM, as is eaſy to ſhew.

As to the other Properties of this Curve, tis evident, from its Conſtruction, that its *Tangent* (as CM) is a *Conſtant Line* every where equal to AR; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve ACB may, for diſtinction, be call'd the *Equitangential Curve.*

7. The Fluxion of the Area $ARMC$ is the little Sector or Triangle MCd, which ſame is alſo the Fluxion of the Sector CDM: whence the Areas $ARMC$, CDM are equal, and the whole Area ACB &c, KMR being infinitely continued, is equal to the Quadrant $AR\alpha$.

8. To find the Radius of Curvature of any Particle, as Cc, from C draw an indefinite Line CT perpendicular to CM, (on the concave ſide of the Curve) and from c another Line perpendicular to cm, which Lines, (becauſe of the Inclination of CM to cm,) will ſomewhere meet as at T, making an Angle $CTc = MCm$. Theſe Angles being equal, their Radii are proportional to their Arcs: therefore, $Md : Cc :: MC : CT$. But $Cc = dm$ (becauſe of $CM = cm$) ſo that $Md : dm$ ($:: CD : DE$) $:: CM : CT$. But $CD = CM$, therefore $CT = DE =$ Tangent of the Arc DM.

9.

9. So that ſuppoſing ATt a Curve Line in which are all the Centers of Curvature of the Particles of ACB, any point as T being found as before, the Length AT (by the nature of *Evolution of Curves*,) is every where equal to the *Tangent* of its correſpondent Circular Arc DM. The Point T is alſo found by making MT perpendicular to RM, and equal to the Secant CE: for ſo is the Angle $CMT = MCD$; and the Triangle MCT equal to the Triangle CDE.

10. Let AHh be an Equilater Hyperbola whoſe Semiaxis is AR and Center R. In the Meridian let RP be equal to the Tangent DE. Join AP, and draw $PH = AP$ and parallel to AR. Compleat the Parallelogram $HNRP$, ſo will the Point H be in the Hyperbola, and its ordinate $HN (= RP = DE = CT)$ be equal to the Curve ATt. From whence, and from *Prop.* 3. *Coroll.* 2. of Dr. *Gregory's Catenaria* (*Phil. Tranſ.* N. 231.) it appears that the Curve ATt is that call'd the *Catenaria* or *Funicularia*, viz. the Curve into whoſe Figure a *ſlack Cord or Chain* naturally diſpoſes its ſelf by the Gravity of its Particles.

" 11. Hence we have another Property of the *Catenaria* " not hitherto taken notice of (that I know of,) *viz.* that " ſuppoſing $AR (= a$, the conſtant Line in Dr. *Gregory*) " equal to the *Radius* of the Nautical Projection, and " RN the Secant of a given Latitude, then is NT the " *Catenaria's* Ordinate at N, equal to RM the Meridio- " nal Part anſwering to the Latitude whoſe Secant is " RN.

12. That TA is the *Catenaria* is alſo demonſtrable from Dr. *Gregory's* firſt *Prop.* Let Tu be the the Fluxion of the Ordinate NT; and $tu (= Nn)$ the Fluxion of the Axe AN. Then becauſe of like Triangles TCM, Tut, $CM : CT (= TA) :: Tu : ut$, that is, as CM a conſtant Line to TA the Curve :: ſo is the Fluxion of the Ordi-

Ordinate, to that of the Axe ($\dot{y} : \dot{x}$) according to *Prop.* 1. *Catenariæ.*

13. From the Premises the Construction and several Properties of the *Catenaria* are easily deducible; one or two of which I'll set down.

1. The Area *ATMR* is equal to *AOPR* a Rectangle contained by Radius *AR* and *RP* the Tangent answering to Secant $HP = TM$. For because of the like Triangles CMm, CEe; $CM : CE :: Mm : Ee$, that is, putting r, s, t, m for Radius, Secant, Tangent and Meridional part *RM*.) $r : s :: \dot{m} : \dot{t}$ whence $r\dot{t} = s\dot{m}$, and all the $r\dot{t}$ = all the $s\dot{m}$, that is $AOPR = ATMR$, which agrees with Dr. *Gregory*'s *Cor.* 5. of *Prop.* 7.

14. Supposing the former Construction, let be added the Line *RH*, including the *Hyperbolic Sector ARH*. I say the same Sector is equal to half the Rectangle *ARMQ* contained by Radius *AR* and the Meridional Part *RM*, ($= \frac{1}{2} r m$), For the Sector ARH = Triangle *RNH* wanting the Semisegment *ANH*. The Fluxion of the Triangle *RNH* is $\frac{s\dot{t} + t\dot{s}}{2}$. The Fluxion of *ANH* is $t\dot{s}$. So the Fluxion of the Sector *ARH* is $\frac{s\dot{t} + t\dot{s}}{2} - t\dot{s} = \frac{s\dot{t} - t\dot{s}}{2}$, 'Tis found before (*Sect.* 13.) that $r : s \left(s : \frac{ss}{r}\right) :: \dot{m} : \dot{t}$; whence $s\dot{t} = \frac{ss}{r}\dot{m}$. And because of the like Triangles CDE, Efe, $CD : DE :: Ef : fe$. But $Ef = Mm = \dot{m}$, because both Ef and Mm are to Md in the same Reason, *viz.* as s to r; therefore $r : t \left(t : \frac{tt}{r}\right) :: \dot{m} : \dot{s}$: whence $t\dot{s} = \frac{tt}{r}\dot{m}$, and $\frac{s\dot{t} - t\dot{s}}{2} =$

2 ss

$\frac{ss - tt}{2r} \dot{m} = \frac{rr}{2r} \dot{m} = \frac{1}{2} r \dot{m}$, = the Fluxion of the Hyperbolic Sector *A R H*, whose flowing Quantity is therefore equal to $\frac{1}{2} rm = \frac{1}{2}$ *A R M Q. Q. E. D.*

15, This shews another Property of the *Catenaria*, viz. that it squares the Hyperbola; for *R M* is equal to *N T* the Ordinate of the *Catenaria*.

16. In *Fig.* 4. Let *A R* be Radius, *A C B* the Equitangential Curve; *M R N* its Asymptote, in which let *M*, *N*, be any two Points equally distant from *R*. Upon *M* draw *M L* parallel to *A R* and equal to the *Difference* of the Secant and Tangent of that Latitude whose Meridional Part is *R M* (by § 3, 4.) Upon *N* draw *N O* parallel to *A R*, and equal to the *Summ* of the foresaid Secant and Tangent. Do thus from as many Points in the Asymptote as is convenient, and a Curve drawn equably through the Points *L - - - A - - - O*, &c. will be a *Logarithmic Curve*, whose *Subtangent* (being constant) is equal to Radius *A R*.

17. Let *n o* be an Ordinate infinitely near and parallal to *N O*. $O p = N n$ the Fluxion of the Asymptote; *O T* the Tangent, and *T N* the Subtangent to the Logarith. Curve in *O*. Then $o p : p O :: O N : N T$. But $O N = s + t$; therefore $o p = \dot{s} + \dot{t}$. $p O = \dot{m}$ (the Fluxion of the Meridian or Asymptote.) So the Analogy is $\dot{s} + \dot{t} : \dot{m} :: s + t : N T$. By *Sect.* 13, 14, $\dot{s} : \dot{m} :: t : r$: also, $\dot{t} : \dot{m} :: s : r$. and thence $\dot{s} + \dot{t} : \dot{m} :: t + s : r$. wherefore is *N T* (the Subtangent to *L A O*) equal to Radius *A R*, a constant Line, and consequently the Curve *L A O* is the Logarithmic Curve, and its Subtangent known.

18. The same Demonstration serves for *L M*, (any Ordinate on the other Side of *A R*) only changing the Sine + into —; and then it agrees with Mr. *James Gregory*'s *Prop.* 3. *pag.* 1[illegible] of his *Exercitations*, viz. *That*

the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their respective Tangents, Radius being Unity. So here RM is the Logarithm of ML, the Difference of the Secant and Tangent of the Latitude whose Meridional part is RM.

19. Supposing the precedent Construction, if through any point C of the Curve ACB be drawn a right Line GCW parallel to MR, terminated with the Logarithmic Curve in W and the Radius AR in G: I say that the same right Line WG is equal to the intercepted part of the Curve Line AC.

20. Let wg be a Line infinitely near and parallel to WG, and terminated by the same Lines; and CS, $W\sigma$, perpendicular to the Meridian; CS intersecting wg in z, and $W\sigma$ in y. Let CM be a Tangent to AC in C; $W\tau$ a Tangent to AW in W; so is $CM = \sigma\tau$. Because of like Triangles Czc, CSM; and Wyw, $W\sigma\tau$; $CS : CM :: Cz : Cc$: also $W\sigma : \sigma\tau :: Wy : yw$. But $W\sigma = CS$; $\sigma\tau = CM$; $Cz = Wy$; therefore is yw the Fluxion of GW, equal to Cc the Fluxion of the Curve AC: Consequently $GW = AC$. *q. e. d.*

It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents standing perpendicular on their Radius. In *Fig.* 3. let $A\gamma\Gamma$ be a Curve whose Ordinates as $g\gamma$, $G\Gamma$, are equal to the Tangents of their respective intercept Arcs Ak, $A\kappa$. Let ΓG be produced to touch the Curve AC in C: then is the Area $A\Gamma G$ equal to the Rectangle contained by Radius AR and GC the produced part of the Ordinate; or $A\Gamma G = AR \times GC$. The Demonstration of which, and of the following *Section*, I for Brevity omit.

22. If we suppose the Figure ACB &c. KR (*Fig.* 3.) infinitely continued, to be turned about its Asymptote RK as an Axe, the Solid so generated will be equal to

a

rectangled Cone whose Altitude is equal to *A R*. And its Curve Surface will be equal to half the Surface of a Globe whose Radius is *A R*. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius *A R*.

The Description of the Rular and Wheel, *Fig*. 2. is sufficient for the Demonstration of the Properties of the Curve: but in order to an actual Construction for Use, I have added *Fig*. 5. where *A B* is a brass Rular; *w h* the little Wheel, which must be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. *s* represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fixt Rular. *p* is a Stud for convenient holding the Rular in its Motion.

Note, *Most of these Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among those of the* Royal Academy *of* Sciences *for the Year* 1712. *but not publish'd till* 1715: *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their Journal.*

VI. *An Account of a Book entituled* Methodus Incrementorum, *Auctore* Brook Taylor, LL.D. & R. S. Secr. *By the Author.*

WHEN I apply'd my self to consider throughly the Nature of the Method of Fluxions, which has justly been the Occasion of so much Glory to its great Inventor Sir *Isaac Newton* our most worthy President, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatise. For it being the Foundation of the Method of Fluxions that the Fluxions

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